# THE FK4 EQUINOX FROM MERIDIAN OBSERVATIONS OF MINOR PLANETS

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## ABSTRACT

Minor planets 1 (Ceres), 2 (Pallas), 3 (Juno), and 4 (Vesta) were observed during a five and a half year period with the Naval Observatory's 7-inch transit circle at El Leoncito, Argentina. From 1043 observations in right ascension and 1020 in declination the correction to the FK4 equinox was found to be $+0''.634 \pm 0''.155$, in good agreement with other determinations of this quantity. This successful equinox determination from a relatively short observing period was possible because the large number of clear nights at El Leoncito permitted many observations to be taken.

## I. INTRODUCTION

The U. S. Naval Observatory's 7-in transit circle observed the minor planets 1 (Ceres), 2 (Pallas), 3 (Juno), and 4 (Vesta) from 1 January 1968 to 21 June 1973 while the instrument was located at the site of the Yale–Columbia Southern Observatory at El Leoncito, Argentina ($\lambda = 4^h37^m19^s.023$ W; $\phi = -31^\circ.802694$), to participate in the international Southern Reference Star (SRS) program. The organization of the observing program at El Leoncito has been described by Hughes (1967), and an analysis of the observational results of the program has been published by Smith (1979). Minor planets were observed with the stars and were treated like stars in Smith's (1979) analysis. The instrumental right ascension ($\alpha$) system was placed on the equinox of the system defined by the Fourth Fundamental Catalog (FK4), prepared by Fricke and Kopff (1963), but the declination ($\delta$) system remains that of the instrument.

## II. TREATMENT OF OBSERVATIONS

During the period that the minor planets were observed, a total of 1044 observations was made, of which 21 were in the $\alpha$ coordinate only. The distribution of the observations among the minor planets is shown in Table I. The mean epoch of the observations was 1970.95 and the mean declination was $+0^\circ.31$.

Before the observations could be used they had to be corrected for the effects of the minor planet's orbital motion during the time it takes to make an observation, about forty seconds. To apply these corrections one needs the rate of change of right ascension, $\dot{\alpha}$, and of declination, $\dot{\delta}$, at the time of observation. These quantities can be obtained by differentiation with respect to time of the usual expression that relates rectangular to spherical coordinates. This results in

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha}\rho\cos\delta \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} \cos\alpha\cos\delta & \sin\alpha\cos\delta. & \sin\delta \\ -\sin\alpha & \cos\alpha & 0 \\ -\cos\alpha\sin\delta & -\sin\alpha\sin\delta & \cos\delta \end{bmatrix} \times \begin{bmatrix} \dot{x}+\dot{X} \\ \dot{y}+\dot{Y} \\ \dot{z}+\dot{Z} \end{bmatrix}, \quad (1)$$

where $\dot{x}$, $\dot{y}$, and $\dot{z}$ are the heliocentric, rectangular velocity components of the minor planet, $\dot{X}$, $\dot{Y}$, $\dot{Z}$ those of the Sun, and $\rho$ is the minor planets geocentric distance. For greatest rigor $\dot{\alpha}$ and $\dot{\delta}$ should be referred to the equinox of date by multiplication of Eq. (1) by the matrices of precession and nutation. The quantity $\dot{\rho}$ is not needed. The velocity components were calculated by numerical differentiation of the rectangular coordinates of minor planets 1, 2, 3, and 4 calculated by Duncombe (1969) and those of the Sun calculated by Herget (1953); $\dot{\alpha}$ was expressed in siderel seconds per solar day and $\dot{\delta}$ in seconds of arc per solar day.

If $\alpha'$ is the right ascension of the minor planet uncorrected for orbital motion and *a*, *b*, and *c* are, respectively, the azimuth, level, and collimation of the transit circle

TABLE I. Number of observations.

| Minor planet | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Observations in $\alpha$ | 333 | 239 | 174 | 298 |
| Observations in $\delta$ | 323 | 238 | 167 | 295 |

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at the instant of observation, then the corrected right ascension is

$$\alpha = \alpha' + \{\dot{\alpha}/(1.002737902 - \dot{\alpha})\}\, a \sin(\phi - \delta) \sec\delta + b \cos(\phi - \delta) \sec\delta + c \sec\delta. \quad (2)$$

The declination corrected for orbital motion is found from

$$\delta = \delta' + \dot{\delta}\Delta\, 0.9972695664 \sec\delta, \quad (3)$$

where $\Delta$ is the difference between the average of the positions of the individual declination measurements and the position of the center of the field of view of the transit circle.

### III. CONDITIONAL EQUATIONS AND SOLUTIONS

To calculate equations of condition for the corrected observations, the procedure outlined in Branham (1978) was followed. To recapitulate this briefly, osculating rectangular coordinates and velocities at epoch JD 2440800.5 (2 August 1970), rather than elliptic orbital elements, were corrected. The partial derivatives in the equations of condition were calculated by the method of Herget (1968). Also included in the equations of condition were partial derivatives for corrections to the Earth's orbit; set VI of Brouwer and Clemence (1961) was used, which solves for: $\Delta l'_0$, a correction to the mean anomaly at epoch of the Earth's orbit; $\Delta\epsilon$, a correction to the obliquity; $\Delta e'$, a correction to the eccentricity; $\Delta\omega'$, a correction to the mean longitude at perihelion; and $\Delta E$, the equinox correction. The last unknown in the equations of condition was $\Delta D$, the equator correction. The cutoff for an observed minus a computed position, $(O - C)$, was taken as $2''.5$. Only one observation in $\alpha$ and three in $\delta$ were excluded by this criterion, which gave 1043 equations of condition in $\alpha$ and 1020 in $\delta$.

Normal equations to solve simultaneously for all of the unknowns were formed in the usual manner. The differential correction procedure had to be iterated twice because the first solution indicated a substantial correction of $\Delta D = -0''.056 \pm 0''.016$ to the fundamentally determined equator, which led to a correction of the declination system defined by the stellar observations (Smith 1979). The solution from the second—and final—differential correction is exhibited in Table II. Normal equations for the contribution from each individual minor planet were also solved and the solutions are shown in Table II. Also exhibited are the mean errors of unit weight, $\sigma(1)$, and the condition numbers for the normal equations. This number is defined as the product of the spectral norms of the matrix of the normal equations, $\boldsymbol{\beta}$, and its inverse.

The correlation matrix for the solution headed "Total" in Table II is 30 by 30 and rather large to display explicitly, but some mention should be made of correlations among the unknowns. There were correlations among a few of the minor planets' osculating rectangular coordinates and velocities and the Earths' orbit, the largest being 87.7%. Among the unknowns in the "Total" column of Table II the only correlation that exceeded the 50% level was one of −58.0% between $-\Delta E \cos\epsilon + \Delta\omega'$ and $\Delta D$.

TABLE II. Solutions for unknowns.

| | Total | 1 |
|---|---|---|
| $\Delta E$ | $0''.634 \pm 0''.155$ | $0''.073 \pm 0''.274$ |
| $\Delta D$ | $0''.004 \pm 0''.016$ | $-0''.018 \pm 0''.030$ |
| $\Delta\epsilon$ | $-0''.115 \pm 0''.053$ | $-0''.265 \pm 0''.092$ |
| $\Delta l'_0$ | $-3''.161 \pm 4''.339$ | $-20''.226 \pm 8''.186$ |
| $\Delta\omega'$ | $3''.384 \pm 4''.362$ | $19''.628 \pm 8''.190$ |
| $\Delta e'$ | $-0''.132 \pm 0''.043$ | $0''.196 \pm 0''.140$ |
| $\sigma(1)$ | $0''.350$ | $0''.328$ |
| cond ($\boldsymbol{\beta}$) | $2 \cdot 10^6$ | $2 \cdot 10^6$ |

| | 2 | 3 | 4 |
|---|---|---|---|
| $\Delta E$ | $1''.034 \pm 0''.562$ | $1''.340 \pm 0''.474$ | $0''.688 \pm 0''.224$ |
| $\Delta D$ | $0''.134 \pm 0''.077$ | $-0''.020 \pm 0''.059$ | $-0''.019 \pm 0''.026$ |
| $\Delta\epsilon$ | $0''.248 \pm 0''.151$ | $0''.003 \pm 0''.140$ | $-0''.222 \pm 0''.083$ |
| $\Delta l'_0$ | $4''.634 \pm 10''.403$ | $2''.088 \pm 12''.549$ | $1''.963 \pm 7''.333$ |
| $\Delta\omega'$ | $-3''.869 \pm 10''.463$ | $-1''.373 \pm 12''.725$ | $-1''.652 \pm 7''.331$ |
| $\Delta e'$ | $0''.110 \pm 0''.145$ | $-0''.235 \pm 0''.248$ | $-0''.220 \pm 0''.127$ |
| $\sigma(1)$ | $0''.389$ | $0''.370$ | $0''.329$ |
| cond ($\boldsymbol{\beta}$) | $1 \cdot 10^6$ | $8 \cdot 10^5$ | $1 \cdot 10^6$ |

### IV. DISCUSSION

The equator correction from the simultaneous solution in Table II is essentially zero and demonstrates that the dynamical equator defined by the minor planets and the fundamentally determined equator from Smiths' (1979) analysis of the star observations are in excellent agreement.

The equinox correction in Table II from the simultaneous solution is a correction to the equinox of the FK4. It is in very good agreement with other determinations of the FK4 equinox, summarized by Fricke (1978), which give values in the vicinity of $0''.75$ at epoch 1970.0.

The small equator correction from the minor planets observed at El Leoncito, which is located at a not too southerly latitude, and the good determination of the FK4 equinox from an observing period of only five and a half years is undoubtedly a consequence of the high percentage of clear nights throughout the program. During the observing period at El Leoncito about 80% of the nights were completely or partially usable. Hence, the number of minor planet observations obtained is roughly twice what it would have been at Washington during the comparable period. In fact, the mean errors of the unknowns in Table II and the concordance of the solutions from the individual minor planets are of the same order of magnitude as those from an extensive series of observations, 2650 in number, of minor planets 1, 2, 3, and 4 analyzed by Jackson (1968). The condition numbers are lower than those for solutions from a series of observations, 10045 in $\alpha$ and 10076 in $\delta$, of minor planets 6, 7, 8, 9, and 15 analyzed by Branham (1978).

This demonstrates that the normal equations in the present investigation are not too ill-conditioned. These three effects, the relatively small mean errors of the unknowns, the concordance of the solutions from individual minor planets, and the lack of extreme ill-conditioning of the normal equations, are consequences of the high density of the observations of minor planets 1, 2, 3, and 4 obtained at El Leoncito.

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